MTH 203: Introduction to Groups and Symmetry

Homework II

(Due 26/08/2022)

1. Establish the assertions in 1.1 (v) of the Lesson Plan.
2. Establish the assertions in 1.3 (ii) (c) & (d) of the Lesson Plan.
3. Let G be a group.
   (a) For $g \in G$, if $g^m = g^n = 1$, for $m, n \in \mathbb{Z}$, then show that $(g)^{\gcd(m,n)} = 1$. [Hint: Use the Euclid's algorithm.]
   (b) Use (a) to establish assertion 1.2 (vi) of the Lesson Plan.
4. Show that every nontrivial group G has a cyclic subgroup.
5. Is $D_{2m} < D_{2n}$, for every $m < n$? Explain why, or why not.